

Effect of Self-Interaction on Vacuum Energy for Yang-Mills System in Kaluza-Klein Theory

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Abstract

We calculate the vacuum energy for Yang–Mills (YM) system in the background space-time $M^4 \times S^3$, taking the effect of self-interaction of the YM fields into account. The compactification scale obtained by Candelas–Weinberg mechanism becomes large if the YM coupling is large. The case with an extra space S^3/Z_2 is also considered, and it is shown that the vacuum associated with broken gauge symmetry is unstable.

In Kaluza-Klein theories,[1] the effect of vacuum polarizations is expected to play important roles in stabilizing the compact space[2] as well as in determining the gauge symmetry in four dimensions.[3]

However, in almost all the calculations of quantum effects in the Kaluza-Klein scheme (except for Ref. [4], only the one-loop approximation is taken into consideration. Thus the calculation carried out so far might not reflect the quantum nature of self-interacting system such as gravitation and YM fields which are inevitably contained in string theory.

The difficulty in going beyond one-loop is caused partly from the lack of appropriate method in computing the higher-loop contribution unambiguously in arbitrary dimensions. In the previous paper,[6] the present authors have developed a technique to handle the YM interaction in vacuum diagrams when the background space-time metric has an extra space S^1 .

It is found that the approximation used in the calculation is more suitable for higher dimensions. We can study the higher-loop effects by our method similarly to the one-loop technique, even if the extra space has a complicated structure. In the present paper, we show the vacuum energy for $SU(3)$ YM system defined in

the background space-time $M^4 \times S^3$, where M^4 is the flat Minkowski space-time and S^3 is the three sphere which has non-vanishing curvature.

We first review our approximation scheme described in Ref. [6]. One can find that the contributions of the vacuum graphs including only YM four-point interaction dominate over the ones of the other graphs of the same order in g^2 (where g is the YM coupling constant) for large dimensionality, D . It arises because the trace of the metric at a closed loop yields a factor D for a vacuum diagram. The graphs which include three-point interactions with derivative couplings are sub-dominant. Therefore it is conceivable that for large D only four-point interaction is important and this simplifies the treatment of YM interactions of higher order. As a check, we have shown in Ref. [6] that the result of the one- and two-loop order of the vacuum energy for YM system in $S^2 \times R^d$ can be reconstructed when $g^2 \ll 1$ by using our method. In our approximation scheme, we start with the YM action where three-point interactions are omitted.

Owing to this simplification, the calculation of the vacuum energy becomes very transparent. We can utilize auxiliary fields [7] to treat the non-linear interaction in the reduced Lagrangian.

A difference from the previous work is the choice of the gauge group. For general gauge group, a quartic interaction which we preserve in the reduced action is

$$\sum_{a,b,c} f^{abc} f^{abc} A_\mu^b A^{b\mu} A_\nu^c A^{c\nu}, \quad (1)$$

where f^{abc} is the structure constant of the gauge group. The other types of interactions are discarded because they cannot produce the graphs of leading contribution (see Ref. [6]). Then we can give the expression of the effective action, in which the gauge fields appear in bilinear form, using auxiliary fields.

Now let us concentrate on the specific case, $SU(3)$ YM theory. Here, we explicitly show the structure of the auxiliary fields.

For concreteness, we use the Gell-Mann's notation.[8] We find

$$\sum_a f^{abc} f^{abc} = \begin{pmatrix} 0 & 1 & 1 & 1/4 & 1/4 & 1/4 & 1/4 & 0 \\ 1 & 0 & 1 & 1/4 & 1/4 & 1/4 & 1/4 & 0 \\ 1 & 1 & 0 & 1/4 & 1/4 & 1/4 & 1/4 & 0 \\ 1/4 & 1/4 & 1/4 & 0 & 1 & 1/4 & 1/4 & 3/4 \\ 1/4 & 1/4 & 1/4 & 1 & 0 & 1/4 & 1/4 & 3/4 \\ 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 0 & 1 & 3/4 \\ 1/4 & 1/4 & 1/4 & 1/4 & 1/4 & 1 & 0 & 3/4 \\ 0 & 0 & 0 & 3/4 & 3/4 & 3/4 & 3/4 & 0 \end{pmatrix}, \quad (2)$$

where the column and row correspond to the suffices b and c . The inverse of

this matrix is

$$M_{ab} \equiv \begin{pmatrix} -1/2 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & -1/6 \\ 1/2 & -1/2 & 1/2 & 0 & 0 & 0 & 0 & -1/6 \\ 1/2 & 1/2 & -1/2 & 0 & 0 & 0 & 0 & -1/6 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & -1/2 & 1/3 \\ 0 & 0 & 0 & 1 & 0 & -1/2 & -1/2 & 1/3 \\ 0 & 0 & 0 & -1/2 & -1/2 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & -1/2 & -1/2 & 1 & 0 & 1/3 \\ -1/6 & -1/6 & -1/6 & 1/3 & 1/3 & 1/3 & 1/3 & -1/2 \end{pmatrix}, \quad (3)$$

and then we use auxiliary fields $\chi^a (a = 1, \dots, 8)$ to get the bilinear form with respect to the YM field. We rewrite the Lagrangian as

$$\frac{1}{2} \sum_{\mu\nu} \sum_a (D_\mu^B a_\nu^a)^2 + \frac{1}{2} \sum_\mu \sum_a \chi^a a_\mu^a a_\mu^a - \frac{1}{4g^2} \sum_{ab} \chi^a M_{ab} \chi^b, \quad (4)$$

where a_μ^a is the quantum fluctuation of the gauge field while D_μ^B means the covariant derivative involving the background classical fields.

Now in order to get the vacuum energy, it is sufficient to find the effective potential $V(\chi)$ in the one-loop approximation within the auxiliary fields. We have

$$V(\chi) = -\frac{1}{8g^2} \chi^a M_{ab} \chi^b - \frac{i}{2(Vol.)} \ln \det(-D_\mu^B D^{B\mu} + \chi^2), \quad (5)$$

where $(Vol.)$ stands for the space-time volume, $\int d^4x$.

In the expression (5), we implicitly take the formal determinant as the one which contains not only transverse and longitudinal components of the vector field but also Faddeev-Popov ghost fields.

The final expression should be obtained from $V(\chi)$ by eliminating the auxiliary field χ with the help of the equation of motion.

The regularization and the calculation technique of the one-loop vacuum energy is the same as Ref. [2, 9, 10, 11]. Particularly, the calculation for vector fields is parallel to Ref. [10, 11].

First we calculate the vacuum energy for $SU(3)$ YM fields in $M^4 \times S^3$, where no background gauge field exists. The symmetry suggests that the auxiliary fields should be set to an equal value, $\chi^1 = \chi^2 = \dots = \chi$. Thus we have to solve one equation for the auxiliary field to obtain the value of the vacuum energy.

The result for the vacuum energy normalized by r^4 (where r is the radius of S^3) is given in Fig. 1. The equation of motion for the auxiliary field χ has been solved by numerical calculation.

In Fig. 1, the vacuum energy slightly increases as a dimensionless combination $\tilde{g}^2 = g^2/r^3$ grows. Obviously, the limit $g \rightarrow 0$ yields the free case, i.e., the vacuum energy for eight species of abelian gauge fields (cf. Ref. [10]).

Next let us suppose the vacuum energy in $M^4 \times S^3/Z_2$. In this case, allowed Kaluza-Klein modes of the YM field are restricted and dependent on the background classical gauge configurations in the extra space.[10]

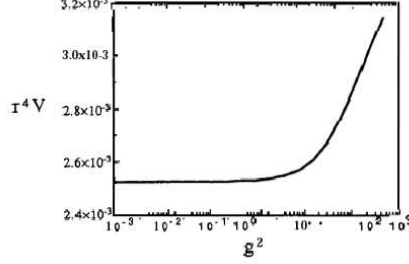


Figure 1: The vacuum energy for the YM system in $M^4 \times S^3$ is plotted against the dimensionless parameter $\tilde{g}^2 = g^2/r^3$.

We consider the following two possible vacuum configurations of gauge fields as the following; one is the trivial case, $\langle A_m \rangle = 0$ and the other is $\langle A_\phi \rangle = \text{diag.}(1, 1, -2)$, where expresses the direction of the one of the azimuthal angle of S^3 (see Ref. [11]). In the vacuum associated with the latter configuration, the four-dimensional gauge symmetry is reduced to $SU(2) \times U(1)$.

Because of the symmetry, we can set $\chi^1 = \chi^2 = \chi^3 = \chi^8 \equiv \chi^A$ and $\chi^4 = \chi^5 = \chi^6 = \chi^7 = \chi^B$ then the quadratic part of the auxiliary fields in the vacuum energy is given as

$$\frac{1}{8g^2}(\chi^A \chi^B). \quad (6)$$

We have to solve two equations of the auxiliary fields simultaneously for each vacuum configuration.

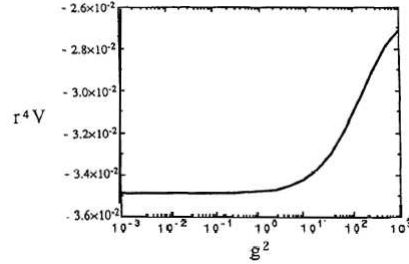


Figure 2: The vacuum energy of the trivial vacuum for the YM system in $M^4 \times S^3/Z_2$, is plotted against the dimensionless parameter $\tilde{g}^2 = g^2/r^3$.

For the trivial vacuum, where we can set $\chi^A = \chi^B$, the results of the numerical calculation for the vacuum energy normalized by r^4 is shown by Fig. 2, similarly to the case of S^3 . In the non-trivial case, however, we cannot find the solution of the equation of motion of auxiliary fields for positive χ^A and χ^B .

This indicates the quantum instability of the non-trivial vacuum, because χ 's stand for “effective squared-masses” of the gauge bosons in physical meaning. (see Eq. (4).)

To summarize, we have computed vacuum energies for $SU(3)$ Yang-Mills theory in partially compactified background geometry in seven dimensions.

The details in the calculation of the vacuum energy will be reported elsewhere.

The vacuum energy for the finite coupling constant in the background $M^4 \times S^3$ is shown in Fig 1. The compactification scheme which utilizes the positive vacuum energy and cosmological constant[2] may not be drastically changed by the self-interaction effect. Of course, the condition for the tuning of the cosmological constant to cancel the four-dimensional vacuum energy would be slightly modified. If $\tilde{g} = 10$ the compactification scale is expected to become about several to ten percent larger than the free-field case.

As for the Hosotani mechanism in the space-time $M^4 \times S^3/Z_2$, it is found that the interaction forces the non-trivial vacuum to be unstable. The self-interaction plays the crucial role in the Hosotani mechanism.

Unfortunately, we cannot advocate the quantum instability of non-trivial vacua for general gauge groups and manifolds. We hope to study the stability of the vacuum in this sense in general self-interacting fields in various dimensions and will report the result elsewhere.

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References

- [1] For reviews, D. Bailin and A. Love, Rep. Prog. Phys. **50**, 1087 (1987); V. M. Emelyanov et al., Phys. Rep. **143**, 1 (1986); I. Ya. Arefeva and I. V. Volovich, Sov. Phys. Usp. **28**, 694 (1985).
- [2] P. Candelas and S. Weinberg, Nucl. Phys. **B237**, 397 (1984).
- [3] Y. Hosotani, Phys. Lett. **B126**, 445 (1983). For more references on Hosotani mechanism, see: K. Shiraishi, Can. J. Phys. **68**, 357 (1990).
- [4] I. L. Bukhbinder, S. D. Odintsov, and O. A. Fonarev, JETP Lett. **51**, 389 (1990); Phys. Lett. **B245**, 365 (1990); Sov. J. Nucl. Phys. **52**, 1101 (1990).
- [5] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring theory*, 2 volumes, (Cambridge Univ. Press, 1987).
- [6] K. Shiraishi and S. Hirenzaki, Z. Phys. **C53**, 91 (1992).

- [7] S. Coleman, R. Jackiw, and H. D. Politzer, Phys. Rev. **D10**, 2491 (1974); S. Coleman, *Aspects of Symmetry* (Cambridge Univ. Press, Cambridge, 1985).
- [8] M. Geil-Mann, Phys. Rev. **16**, 1067 (1962).
- [9] T. Appelquist and A. Chodos, Phys. Rev. **D28**, 772 (1983); T. Inami and O. Yasuda, Phys. Lett. **B133**, 180 (1983).
- [10] Y. Kato and J. Saito, Prog. Theor. Phys. **74**, 1145 (1985); O. Foda, preprint IC/84/238 (December 1984), unpublished; E. J. Copeland and D. J. Toms, Nucl. Phys. **B255**, 201 (1985); I. H. Russell and D. J. Toms, Class. Quantum Grav. **4**, 1357 (1987).
- [11] K. Shiraishi, Prog. Theor. Phys. **78**, 535 (1987); A. Nakamura and K. Shiraishi, Phys. Lett. **B215**, 551 (1988); J. S. Dowker and S. P. Jadhav, Phys. Rev. **D39**, 118 (1989); *ibid.* **D39**, 2368 (1989).